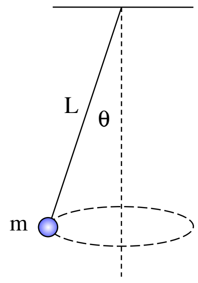
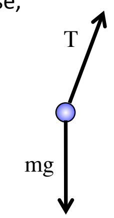
Problem 6.8

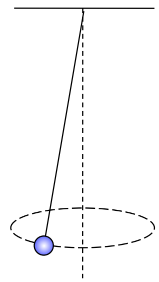
This is a good problem because it makes you think about the kinds of things that can be confusing when dealing with centripetal force situations. Specifically: 1.) how do you orient your f.b.d., and 2.) how do you determine the "centripetal" direction? To those ends:



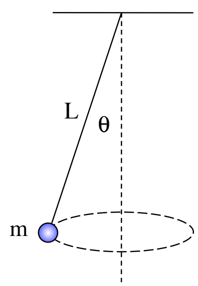
1.) You need to be able to see all the pertinent forces acting in the system when conjuring up your f.b.d. In most cases, as in this case, you want to look at the body *as it is approaching you*. That f.b.d. is shown to the right.



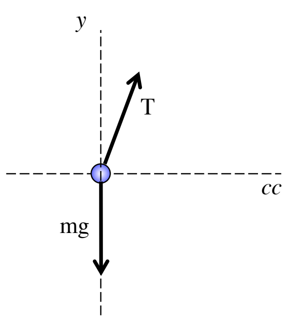
NOTICE that the instructions for determine the f.b.d.'s orientation would have worked equally well if the sketch had looked like the one shown to the far right.



2.) Reiterating what was said in *Problem 6.1* about how to determine the "center seeking (centripetal) direction:" Identify the center of the arc upon which the body is moving. In this case, that will be to the right of the ball, horizontally out (versus along the line of the tension . . .). Once identified, draw an axis from the body through that center. That will be the centripetal direction. Once done, draw an axis perpendicular to the centripetal axis. Doing all that for our problem yields the f.b.d. shown below:

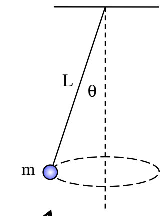


Note that the "cc" on the horizontal axis stands for "center seeking." Clearly I was lousy at spelling as a kid. That, or it's for "centripetal" as said by someone who repeats themselves. And no, this is not a slur. *I* repeat myself . . . often (just ask Mr. White about my story telling propensities.)!

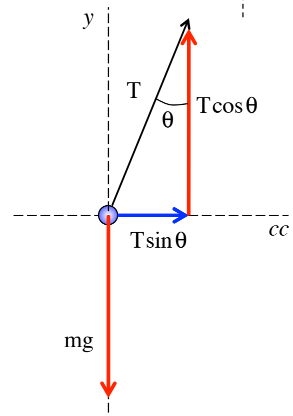


toward center of arc

Once you have the f.b.d. and have set the axis with the help of the question, "What is the centripetal direction?", the problem becomes a standard N.S.L. problem with the modification that there is a non-zero acceleration (in fact, it's equal to mv^2/R)--a centripetal one--in the direction PERPENDICULAR to the direction of motion (i.e., PERPENDICULAR to the velocity vector).



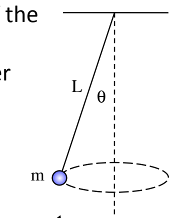
Soooo, next step is to note that there are off-axis forces acting. Breaking those into components yields the sketch to the right:



At this point, you can get the vertical component by inspection. In fact:

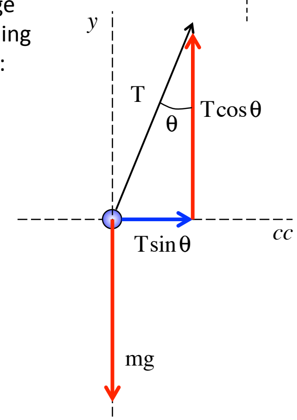
$$\begin{aligned} T_y &= mg \\ &= (80.0 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 784 \text{ N} \end{aligned}$$

The problem is that we can't simply state that the magnitude of the horizontal component of "T" will be equal to "mv^2/R" (which is actually the case) because we don't know "v." We need another relationship. To get it, we'll use N.S.L.:



The modified f.b.d. for the situation is shown below.

Remembering that the bob is moving out of the page toward you (that is how we set up the f.b.d.), summing the forces in the vertical with no acceleration yields:



$$\sum F_y :$$

$$\Rightarrow \quad T\cos\theta - mg = m\cancel{a_y}^{\,0}$$

$$\Rightarrow \quad T\cos\theta = mg$$

$$= 784 \text{ N} \quad (= T_y)$$

as deduced by inspection earlier,

AND:

$$T\sin\theta = 784 \text{ N}$$

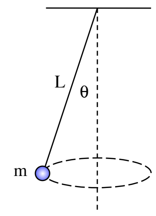
$$\Rightarrow \quad T = \frac{784 \text{ N}}{\cos 5^\circ} = 787 \text{ N}$$

In the “center-seeking” direction, we can write:

$$\sum F_c :$$

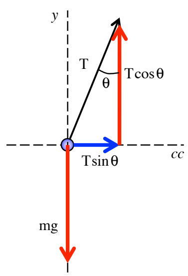
$$\Rightarrow \quad T\sin\theta = ma_c$$

$$\Rightarrow \quad T\sin\theta = m\left(\frac{v^2}{R}\right) \qquad (= T_x)$$



This would end it if we knew “mv^2/R“ or “$T\sin\theta$.” Fortunately, we can determine the latter:

$$T_x = T\sin\theta$$
$$= (787 \text{ N})\sin 5^\circ$$
$$= 68.6 \text{ N}$$



In other words:

$$\vec{T} = (68.6 \text{ N})\hat{i} + (784 \text{ N})\hat{j}$$

b.) The radial (centripetal) acceleration?

Again, it would be tempting to write out "v^2/R," but we still haven’t determined “v” or, for that matter, “R” (though “R” is just “$L\sin\theta$“). Instead, we will use the uninspiring (but correct):

$$T\sin\theta = ma_c$$

$$\Rightarrow \quad a = \frac{T\sin\theta}{m}$$

$$= \frac{(787 \text{ N})\sin\theta}{(80.0 \text{ kg})}$$

$$= .857 \text{ m/s}^2$$

This acceleration is toward the center of the circle.